REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE

3. REPORT TYPE AND DATES COVERED
FINAL/15 NOV 92 TO 14 APR 95

4. TITLE AND SUBTITLE
DISCRETE, STOCHASTIC,AND OPTIMIZATION APPROACHES TO PROBLEMS OF NETWORKS AND SCHEDULING

5. FUNDING NUMBERS
2304/DS
F49620-93-1-0041

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AFOSR-TR-95
0673

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
110 DUNCAN AVE, SUTE B115
BOLLING AFB DC 20332-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
F49620-93-1-0041

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

New results have been obtained that nible to the design of efficient, reliable and invulnerable networks.

19951018 059

14. SUBJECT TERMS

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR(SAME AS REPORT)

THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

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FINAL TECHNICAL REPORT

TO: AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

DISCRETE, STOCHASTIC, AND OPTIMIZATION APPROACHES TO PROBLEMS OF NETWORKS AND SCHEDULING

GRANT NUMBER F49620–93–1–0041

ACCOMPLISHMENTS: November 15, 1992 – March 14, 1995

Peter L. Hammer
Co–Principal Investigator

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Co–Principal Investigator

September 8, 1995

RUTCOR

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This summary of research accomplishments is organized into essentially the same sections as is our original proposal. Papers referred to by number are listed below in the list of publications prepared under the grant. Papers referred to with authors' names and year are listed at the end of the section.

1. Overview of the Project and Applied Motivation

In this project, we have done research in a number of areas of discrete mathematics which are related to networks and scheduling. The research problems considered were motivated by a number of practical problems which are of interest and importance to the Air Force.

The Air Force problems which motivated this project involve:

*design of routing networks
*location of hubs, points of embarkation, staging areas, and other facilities
*problems of scheduling involving load allocation, route assignment, point of embarkation assignment, and crew assignment
*issues of command, control, and communications
*the need to introduce stochastic elements into models which have traditionally been treated by deterministic methods
*the need to deal with problems in an on–line basis, without waiting for complete information.

The ***network design problems*** we have been concerned with are motivated by problems of design and analysis of routing networks such as those used by the Military Airlift Command (MAC)/Air Mobility Command (AMC). Among the multiple goals for such networks are that they be efficient, inexpensive, reliable, and invulnerable. These goals are very similar to the goals which have been traditionally set forth for the networks of command, control, and communications (CCC). (See for example Bracken [1983].) We have studied network concepts of efficiency, reliability, and vulnerability, motivated by the need to evaluate such network decisions as the decision to design a system using a hub–and–spoke design which has been adopted by AMC. We have also kept the applications to CCC networks in mind, as well as benefiting from the extensive literature on efficiency, reliability, and vulnerability of CCC systems.

In many problems of the Air Force, one is interested in ***locating*** a facility where it is convenient to a group of users. This is particularly true in the case of a number of the problems of interest to AMC. For instance, methods of operations research have been used at AMC and the Air Force Logistics Center (AFLC), through the Optimal Airlift Distribution System (OADS) model, to locate U.S. hubs at Travis Air Force Base in California and Tinker Air Force Base in

Oklahoma. They have also been used to identify good points of embarkation in deliberate planning models, to identify staging areas for medical evacuations, and to identify hubs for the defense courier system. Similar problems arise more generally in any situation where we need to locate a facility which is conveniently located to a number of "users." The theory of facility location has been developed to handle such problems, and we have studied a variety of questions in location theory.

Many Air Force activities involve ***scheduling problems***. For instance, at AMC, scheduling problems arise in allocating loads to airplanes and to points of embarkation or hubs, assigning loads to routes, assigning crews to airplanes, and so on. These problems have numerous complications with which the state of the art in scheduling theory is not equipped to deal. We have developed the theory of scheduling as it relates to such Air Force needs as taking account of user preferences for schedules (such as unit integrity constraints imposed by commanders), finding schedules which meet performance standards (such as embodied in the UMMIPS priorities at AMC), and finding schedules which are stable under modifications or disruptions.

When it comes time to make decisions in a crisis, the success or failure of a national defense effort depends critically on the ***command, control, and communications*** (CCC) system. Such systems must, like routing networks, be efficient, invulnerable, and reliable. In developing the Mobility Analysis Support System (MASS) in connection with Operations Desert Shield and Desert Storm, AMC found that issues of command and control were important logistics factors which were difficult to measure and model, and new concepts of modelling them are needed for future versions of this and other models. In this project, we have studied the vulnerability and reliability of CCC systems, as well as considered other aspects of modelling their design and use.

Many of the problems of the Air Force, in particular problems studied at AMC, have complicated ***stochastic components*** (involving weather, equipment failure, etc.), but have heretofore been modelled in the Air Force by using methods of deterministic optimization or only crude methods of stochastic optimization. The desire to develop stochastic approaches to problems is a recurrent theme in this project. It has been a goal of this project to develop stochastic approaches to network design problems, location problems, and scheduling problems. In addition, we have devoted a considerable amount of time to one particular aspect of research on stochastic methods, the development of a theory of stochastic optimization under integral decision variables.

In analysis of practical problems with methods of operations research, there is increasing emphasis on finding solution algorithms which are ***on–line*** in the sense that one is forced to make choices at the time data becomes available, rather than after having the entire problem spelled out. A large effort, the U.S. Transcom/DARPA Planning and Scheduling Initiative, is being devoted to the development of a highly interactive strategic mobility modelling tool which will allow an Air Force user to get detailed information about many aspects of the Air Force transportation systems and make on–line decisions. The emphasis on such a modelling tool underlines the importance of on–line methods for current Air Force problems. While much Air Force planning is of a "deliberate" nature, deliberate plans need to be changed in an on–line manner during "execution" planning. We have sought to develop on–line methods for dealing with problems of network design, location, and scheduling, and we have devoted some attention to the theory of on–line methods in general.

In many cases, the Air Force problems described here call for the development of totally new concepts of mathematics or operations research, and for the development of new models which capture a notion of importance for an application. Thus, in addition to doing research on specific mathematical and computational questions, which has involved the majority of our time, we have devoted considerable effort in this project to ***model–building***, and to preparing precise formulations of certain problems which, in our early thinking, were described in relatively imprecise language.

The next four sections each describe a particular set of mathematical problems and relate them to the motivating applied problems. These sections deal with design of efficient, reliable, and invulnerable networks; location problems; scheduling problems; and methods for solving stochastic integer programming problems.

2. Design of Efficient, Reliable, and Invulnerable Networks

Networks of all kinds, electrical networks, computer networks, and in particular routing networks, differ in terms of their efficiency, reliability, and vulnerability. In fact, there are many different ways to define these terms, and in many cases the corresponding concepts are not easy to compute. Moreover, in many cases we face a tradeoff among different goals, such as maximizing reliability and minimizing cost. Also, our intuition is not always very good as to the relationships among these various goals. For instance, Cohen [1988] shows that increasing the "redundancy" in a routing or command and control network does not always increase its reliability, contrary to widespread intuition. There is the need for a great deal of research to develop concepts of efficiency, reliability, and vulnerability, to learn how they relate to each other, and to find effective ways to utilize them in practice, and this project has concerned itself with this topic.

Let us concentrate momentarily on the notion of reliability. We use this concept somewhat interchangeably with the concept of invulnerability. A recent general reference on the subject of reliability in networks is the book by Hwang, Monma, and Roberts [1991]. Networks are subject to failures of different components. If a network is modelled as a graph, then most of the models of network reliability study failures of either vertices or edges. In particular, in an aircraft routing network, airfields can be closed down either permanently due to damage or temporarily due to weather or overcrowding or lack of certain kinds of facilities such as gates, unloading equipment, ground crews, or fuel. These are vertex failures. There are also edge failures due to weather, payload capacity over–runs, and political factors (e.g., flyovers of different countries' airspace). There are several fundamental questions in network reliability theory: How reliable is a given network? How do we design a network which is optimally reliable given certain constraints? How should a network be designed so as to make disruption by an accidental or deliberate failure as difficult as possible? How "stable" is a network in the sense that we can reconstruct it or reconfigure it (relatively quickly or cheaply or in an on–line manner) so as to achieve a network which meets similar constraints? In this project, we have considered these types of questions. The answers to these questions, and similar questions involving efficiency and vulnerability, are dependent of course on how we define these concepts. Thus, some of our research has involved investigation of different concepts of vulnerability and reliability and the development of new concepts.

Our research on networks has involved three main directions: investigation of concepts of vulnerability, efficient network design, and measurement of the reliability of a network under stochastic failures.

We have looked at alternative concepts of vulnerability. For a long time, there has been a great deal of work devoted to defining alternative concepts of vulnerability of a network. The literature on this subject is summarized in Moazzami [1993], Bagga, et al. [1993], and Barefoot, Entringer, and Swart [1987]. In this project, we have both developed new concepts and analyzed old ones. While much is still to be learned about those concepts which have already been defined, there is also a need for the development of new concepts appropriate for new applications. As we have noted above, some measures of vulnerability are related to the problem of reconstructing a damaged network. To define some of these, let us take $t(H)$ to be the number of vertices in the largest component of network H and $c(H)$ to be the number of components of H. Then the ***vertex integrity*** of G is the minimum value of $|S| + t(G-S)$ over all subsets S of vertices; the ***edge integrity*** of G is the minimum value of $|F| + t(G-F)$ over all subsets F of edges of G; the ***vertex (edge) toughness*** of G is the minimum value of $|A|/c(G-A)$ over all subsets A of vertices (edges) of G; and the ***vertex (edge) tenacity*** of G is the minimum of $[|A| + t(G-A)]/c(G-A)$ over all subsets A of vertices (edges) of G. We have concentrated on the concept of edge tenacity. The stability of a network composed of (processing) nodes and (communication or transportation) links is related to two concepts: How many nodes can still communicate after the loss of links or nodes and how difficult is it to reconnect the network? The edge tenacity is a measure of this stability that considers the tradeoff between the number of components of a network after part of it has been destroyed, which measures the difficulty of reconnection and can be thought of as an "attacker's" "reward" or "payoff"; and the "cost" to an "attacker" as measured by the size of the destroyed set of links and the size of the largest remaining component, since a larger remaining component means the "attack" was not quite as successful. Vertex tenacity is studied in unpublished work of Cozzens and Stueckle and in theses by Moazzami [1993] and Mann [1993]. We know of no prior work on edge tenacity. In paper [50], we have studied the ***edge–tenacious networks***, those in which the edge tenacity is achieved by destroying all of the edges. These networks can be considered very stable because to minimize the ratio of cost of an attack to reward of making such an attack, an attacker needs to destroy all of the edges. Thus, attacks tend to be "expensive" and so the networks are relatively invulnerable. Surprisingly, we were able to show that many of the network topologies used to design highly reliable computer, communication and transportation networks are edge–tenacious. The topologies for which we have demonstrated this result include the r–regular, r–edge–connected graphs, and in particular complete graphs, complete multipartite graphs $K(m,m,\ldots,m)$, powers of cycles, n–cubes, inflations, complete bipartite graphs, and the "Harary graphs."

Edge–tenacious networks are highly reliable and "efficient" networks in the sense of stability and invulnerability to "attack." We have investigated other notions of efficiency for networks as well. In particular, we have been concerned with problems of orientation of networks. To route traffic efficiently, it is sometimes necessary to make certain routes one–way. A long literature is concerned with orientations of networks that result in strongly connected digraphs, a minimum requirement. In this literature, various concepts of efficiency are considered, and the question of how to find the most efficient orientation is

analyzed. See Roberts and Xu [1994] for a summary of references on this topic. We have analyzed the notion of efficiency that involves minimizing the directed diameter of the resulting directed network. Specifically, building on work begun in an earlier project, we have analyzed networks whose undirected topology involves an annular structure with concentric traffic rings and a hub joined by spokes heading to the outer rings from the center. In the paper [5], we have identified optimal orientations for these annular networks in the sense of directed diameter minimization.

A *switchbox* is a rectangular grid of horizontal tracks and vertical columns. Such rectangular grids are important network design concepts. A ***net*** is a collection of boundary points of such a switchbox and a ***switchbox routing problem*** is a set of pairwise disjoint nets. The solution of such a routing problem is the realization of the nets as pairwise vertex disjoint subgraphs (usually Steiner trees) of the planar grid graph so that each subgraph connects the boundary points of the net. In paper [20], we consider the gradually more and more complex problems of switchbox routing called single row routing and channel routing, and the gradually less and less restrictive models (involving different assumptions about the realization) called 1–layer, Manhattan, unconstrained 2–layer, and multi–layer. The single row routing problem can always be solved in the Manhattan model and the channel routing problem can always be solved in the unconstrained 2–layer model, in fact both in linear time. In paper [20] we show that the general switchbox routing problem is solvable in the multilayer model, also in linear time.

A ***marked graph*** is a graph in which each vertex is given a sign, + or –. We call a marked graph ***consistent*** if every cycle has an even number of – signs. Marked graphs are commonly thought of as models of communication networks in which binary messages are sent through the network and vertices with sign – correspond to places at which messages are reversed. A consistent marked graph has the important consistency property that if a message is sent from x to y along two different paths, y will receive the same message no matter which path is followed. The notion of consistency also arises in social networks and in the study of balanced signed graphs which have important applications in energy modelling and discrete optimization. The problem of characterizing consistent marked graphs was solved by Beineke and Harary [1978] and has since been studied by a variety of authors interested in giving good recognition algorithms. However, the problem of characterizing unmarked graphs that can be consistently marked with at least one – sign has remained open since 1978. In paper [61] we find a partial solution to this problem, by describing all the markable blocks with no cycle of length greater than 5. In paper [69] we present several conditions, algorithms, and structural characterizations for consistent marked graphs. We also present some graphs that can be consistently marked with at least one – sign and others that cannot, and give a polynomial time algorithm to check if a graph can be so marked. In paper [75], we study the structure of 2–connected graphs, and in particular we note that in a marked graph, each 3–connected vertex pair has the same sign if and only if for every spanning tree, each pair of fundamental cycles has end vertices with the same sign. The set of fundamental cycles with respect to a spanning tree of graph G forms a basis for the cycle space of G. In paper [71], we investigate such cycle bases, and give an algorithm for finding all cycle bases of a graph. We also investigate cycle bases for planar graphs that consist of inner bases, calling such a cycle basis a ***face basis***.

Our work on consistent marked graphs led us back to consider balanced signed graphs. In paper [73], we introduce a notion of restricted balance for

signed bipartite graphs and a notion of induced consistency for marked graphs, and show that a signed bipartite graph G is balanced if and only if under a certain associated marking, G is induced–consistent as a marked graph. Moreover, G is restricted balanced if and only if under the associated marking, G is consistent as a marked graph. In paper [74], we analyze one measure of degree of balance, the *line index* or smallest number of edges whose sign change leads to balance, and more generally to the line index for a weighted signed graph. We show that the problem of computing the line index (for a weighted signed graph) is equivalent to the problem of finding the capacity of the minimum cut of the same graph and that there is a polynomial time algorithm for finding the line index (for weighted signed graphs). The capacity of the minimum cut of a graph is a widely studied measure of reliability of the graph. Orlova and Dorfman and also Hadlock have given a polynomial algorithm for finding a min cut when all weights are negative and the graph is planar. In paper [70], we generalize that algorithm to find a min cut when weights are any real numbers and the graph is planar. We also give a linear algorithm to find min cut for series–parallel graphs. The thesis [72] contains the results of papers [69,70,71,73,74,75].

Among the more intriguing candidates for possible efficient network design are topologies arising in various natural physical and chemical systems. In paper [41], we study the topologies involving finite connected subgraphs of the infinite hexagonal lattice which has no nonhexagonal interior faces or cut edges. Although our emphasis here is on the chemical applications, where these topologies are called benzenoid systems, we hope that our emphasis on various topological indices such as the Clar number might lead to results of interest for design of good transportation and communication networks.

A problem that arises in the efficient design of networks is the ***Steiner tree problem***, which asks for a shortest network connecting n terminals. The ***rectilinear Steiner tree problem*** is the Steiner tree problem when the terminals are in the rectilinear plane and we use the L_1 metric. The problem can be considered a special case of the Steiner tree problem in a grid network obtained by drawing horizontal and vertical lines through terminals. The reductions developed for the network variant of the Steiner tree problem can therefore also be used in the rectilinear case. However, computational experience indicates that they are not very powerful in the rectilinear grids. In paper [65], we introduce a limited number of rectilinear reductions based on the geometrical properties of terminals. Experimental results show that these reductions are extremely powerful while being quite simple to verify and easy to implement. For randomly generated problem instances with one terminal per row and column, the number of non–terminals remaining seems to be around 20 to 25 percent of their original number. Furthermore, the performance of geometrical reductions improves as the density of terminals in fixed–size grids grows. In papers [64,66], we study a Euclidean Steiner tree problem with obstacles. Here we are in the Euclidean plane and we have some obstacles blocking the possible solutions. Concatenation of Steiner minimal trees for well–chosen subsets with up to four terminals leads to good quality solutions. Polynomial algorithms for three and four terminals inside a simple polygon are developed. Analogous work on a Euclidean traveling salesman problem with obstacles is described in Section 4.

Among the candidates that have been widely studied as potentially highly efficient networks are those arising from circulant graphs and circulant matrices and their powers. We have studied powers of circulants in paper [1]. In particular, in bottleneck algebra (where addition and multiplication are replaced by the max and

min operations), we consider the powers of a square matrix A. These powers are periodic, starting from a certain power A^k. The smallest such k is called the exponent of A and the length of the period is called the index of A. Cechlarova has characterized the matrices of index 1. We consider circulant matrices, and determine when such matrices are idempotent (have exponent and period equal to 1). When the index is 1, we say that the circulant is strongly stable, and we show when this happens and observe that the result is equivalent to the result of Cechlarova for the case of circulant matrices.

We have explored the connection between network design and VLSI design in paper [17]. There, we consider cell flipping in VLSI design, an operation in which some of the cells are replaced with their "mirror images" with respect to a vertical axis, while keeping them in the same slot. After the placement of all cells, one can apply cell flipping in order to decrease the total area, approximating this objective by minimizing total wire length, channel width, etc. However, finding an optimal set of cells to be flipped is usually a difficult problem. In this paper, we show that cell flipping can be efficiently applied to minimize channel density in the standard cell technology. We prove that an optimal flipping pattern can be found in $O(p(n/c)^c)$ time, where n, p, and c denote the number of nets, pins, and channels, respectively. Moreover, we show that in the one channel case (i.e., where $c = 1$), the cell flipping problem can be solved in $O(p \log n)$ time.

In modeling various concepts in the social sciences, we use linear "structural equation" models, of which models of factor analysis, path models, and random regression models are all special cases. In such a model, it is assumed that there is a set of variables, for each there is a unique associated error term, and linear equations relate variables to sets of others and error terms. One of the reasons we are interested in such linear structural equation models is that data about network performance can be modeled using such models. Unfortunately, linear structural equation models might not fit such data or other data exactly. One common reason for this is that a model can entail a constraint on the correlation matrix which does not hold in the data. It is possible to improve the fit of such a model by modifying it so that it entails only the constraints that (approximately) hold in the data. In paper [63], we are concerned with the question of finding rank constraints on the correlation matrix, and in particular we emphasize the rank constraints that correspond to so–called tetrad differences. We find methods for modifying linear models that are necessary and sufficient to eliminate constraints concerning ranks higher than 1. The methods can strengthen procedures used to search for structural equation models for large data sets arising not only in network design but also in many areas of the social and natural sciences.

The analysis we have discussed so far does not consider probabilities of failure. In the traditional probabilistic approach to network reliability, we assume that each edge or each vertex fails with a given probability. It is often reasonable to assume that these failures are statistically independent. However, during a crisis, there will be common causes which will result in failure of the edges being statistically dependent. These dependencies will greatly affect all computations and the modelling of the dependencies is a major research challenge. We have devoted a considerable amount of effort to analysis of network reliability under probabilistic failure assumptions. In paper [55] we obtain lower and upper bounds for the probability that at least one of a finite number of events occurs given that the individual and the joint probabilities of any two events are known. The bounds improve on the existing lower and upper bounds and can be used to approximate

reliability of complex series–parallel systems. A linear programming problem is used to obtain the bounds and the bounding technique allows for exploiting all available information.

Paper [56] presents new bounds on logical functions of events such as network failure where the input data are joint probabilities of at most m (out of n) events. Among the bounds presented in the paper are those which are created by a method of multivariate polynomials.

One area of particular interest in the analysis of networks with failures is to analyze the causes of network failures. We have studied the problem of understanding cause–effect relationships from incomplete observations. We consider n different events and an effect that is present when some of these events have occurred. We try to understand on the basis of some observations exactly when a combination of events leads to the effect. Specifically, we have been concerned with n boolean variables, $x_1, \ldots, x_n$, corresponding to the event. We have N observations in which some of these x_i's are 1 (the corresponding event occurred) and the desired effect took place or the corresponding effect did not take place. We try to fit a linear model with noise. Specifically, we look for parameters $a_1, \ldots, a_n$, a random variable u, and a threshold value y_0, so that $y = a_1x_1 + \ldots + a_nx_n + u$ is larger than y_0 if and only if the effect took place. This approach was motivated by a paper of Crama, Hammer, and Ibaraki [1988] in which a nonstochastic cause–effect relationship was formulated. In terms of networks, we note that if certain sets of edges fail, the network fails, and if other sets of edges fail, the network doesn't fail. We then try to assign weights a_i to the event that edge i fails so that failure of the network corresponds to the weighted sum of the edge failures being over threshold. In paper [52], we formulate the stochastic version of the problem, with the noise parameter u, as a specially structured stochastic linear programming problem and solve it by a dual type linear programming algorithm.

Paper [15] is concerned with a different variant of the non–stochastic cause–effect model of Crama, Hammer, and Ibaraki [1998]. We model the problem as that of predicting the value of a discrete function on the basis of a set of observations that is incomplete in two senses. Specifically, we observe the values of only some of the arguments of the function, and we observe the function value only for certain combinations of these arguments. Our goal is to predict the value of the function for any combination of values for the arguments. We address the problem under a monotonicity condition that is natural in many applications, and we display a special class of problems for which the best monotone prediction can be found in polynomial time.

Partially or incompletely defined discrete functions are also studied in paper [13]. Here, we consider the problem of decomposability of such functions. The results include polynomial time algorithms for certain important types of decomposition, as well as NP–completeness proofs for more complex structures.

In another paper on partially defined discrete or boolean functions, paper [19], we address a fundamental problem related to the induction of boolean Logic: Given a set of data, represented as a set of binary "true n–vectors" (or "positive examples") and a set of "false n–vectors" (or "negative examples"), establish a boolean function (extension) f with some specified properties so that f is true (respectively false) in every given true (respectively false) vector. We study this problem in the presence of some a priori knowledge about the extension f. Such

knowledge may be obtained from experience or from the analysis of mechanisms that may or may not cause the phenomena under consideration. The real–world data may contain errors, e.g., measurement errors might come in when obtaining data, or there may be some other influential factors not represented as variables in the vectors. To cope with such situations, we may have to give up the goal of establishing an extension that is perfectly consistent with the given data. If there is no such extension, the best we can expect is to establish an extension f which has the minimum number of misclassifications. Both problems, i.e., the problem of finding an extension within a specified class of boolean functions and the problem of finding a minimum error extension in that class, are extensively studied in paper [19]. For certain classes, we provide polynomial algorithms, and for other cases we prove their NP–hardness.

In paper [16], we consider the problem of identifying an unknown boolean function f by asking an oracle the functional values $f(a)$ for a selected set of test vectors a in $\{0,1\}^n$. Furthermore, we assume that f is a positive (or monotone) function of n variables. It is not known yet whether or not the whole task of generating test vectors and checking if the identification is completed can be carried out in polynomial time in n and m, where $m = |\min T(f)| + |\max F(f)|$ and $\min T(f)$ (respectively $\max F(f)$) denotes the set of minimal true (respectively, maximal false) vectors of f. To partially answer this question, we propose in this paper two polynomial time algorithms that, given an unknown positive function f of n variables, decide whether or not f is 2–monotonic, and if f is 2–monotonic, output both sets $\min T(f)$ and $\max F(f)$. The first algorithm uses $O(nm^2+n^2m)$ time and $O(nm)$ queries, while the second uses $O(n^3m)$ time and $O(n^3m)$ queries.

In one model of network reliability based on edge failures, a ***state*** of a network at any point in time is the set of links which are operating, and a state is called ***operational*** if the system meets certain kinds of standards when the network is in that state. The simplest notion of being operational is that every pair of vertices in a set K is connected by an operating path in that state. The problem of computing the probability that the system is operational is then called the ***K–terminal reliability problem***; when $|K| = 2$ it is the ***2–terminal reliability problem*** and when K is the set of all vertices it is called the ***all–terminal reliability problem***. The probability of being operational is often called the ***reliability of the network***. The K–terminal reliability problem has been widely studied, especially in connection with CCC networks. See for example Colbourn [1987], and Colbourn and Litvak [1991]. Even though the K–terminal reliability problem is already a dramatic oversimplification of real reliability problems, it is already #P–complete even when $|K| = 2$ and all edges have the same probability of operation. All known exact methods for solving it even appear to have exponential time average case running time. Because of this fact, attention has been turned to finding methods for bounding reliability. Probabilistic logic is closely connected with this problem. A basic model for probabilistic logic, outlined by Boole [1854], has been developed in Hailperin [1965,1986] and Nilsson [1986]. Zemel [1982] has shown how it could be used to find best possible bounds for 2–terminal reliability without an independence assumption. Another method for finding approximations based on the probabilistic logic methods of Hailperin, Nilsson, and Zemel was proposed in Boros and Prékopa [1989]. They use linear programming methods to obtain bounds on probabilities. A small size relaxation of the originally huge linear programming problem is considered and sharp bounds are computed from the small LP. This technique is applied successfully to bound the reliability of binary systems given explicitly by disjunctive normal formulae. With

this background, we have concentrated on the development of probabilistic logic. A central facet of probabilistic logic is the boolean satisfiability problem, which is central to combinatorial algorithms. Let V be a set of n boolean variables and V' the set of boolean complements of these variables. The elements of $L = V \cup V'$ are called ***literals***. Given a boolean formula in conjunctive normal form (CNF), the ***satisfiability problem*** consists of finding a satisfying true/false assignment to the variables or in recognizing that no such assignment exists. A ***partial assignment*** is a subset S of literals so that $S \cap S' = \phi$. A ***Horn formula*** is a boolean formula on these n variables so that for all partial assignments L, $|L \cap V'| \leq 1$. Horn formulae play a prominent role in artificial intelligence and logic programming. Their importance is due to a large extent to the fact that for such expressions the satisfiability problem is linearly solvable. We have analyzed the problem of compression of knowledge bases, which is the problem of boolean function minimization In paper [35], we investigate this problem for the class of Horn production rule knowledge bases. The standard approach to this problem, consisting of the removal of redundant rules from a knowledge base, leads to an "irredundant" but not necessarily optimal knowledge base. We prove here that the number of rules in any irredundant Horn knowledge base involving n propositional variables is at most $n-1$ times the minimum possible number of rules. In order to formalize the optimal compression problem, we define a boolean function of a knowledge base as being the function whose set of true points is the set of models of the knowledge base. In this way, the optimal compression of production rule knowledge bases becomes a problem of boolean function minimization. In this paper we prove that the minimization of Horn functions (i.e., boolean functions associated to Horn knowledge bases) is NP–complete. The problem of minimizing a boolean formula amounts to finding its equivalent representation which has the minimum possible number of terms. In paper [7] we show that, given a Horn formula, finding a disjunctive normal form (DNF) equivalent to it and having the minimum possible number of terms is NP–complete. Paper [36] deals with the minimization of quasi–acyclic Horn functions, the class of which properly includes the two practically significant classes of quadratic and of acyclic functions, as noted above. A procedure is developed for recognizing in quadratic time the quasi–acyclicity of a function given by a Horn CNF, and a graph–based algorithm is proposed for the quadratic time minimization of quasi–acyclic Horn functions. Papers [34] and [33] deal with logic minimization for expert systems and graph–based methods for Horn knowledge compression, respectively.

Other work on Horn functions and other boolean functions is in papers [32], [21], [28], [23], [6], [12], and [39]. In paper [28], we provide a simple characterization of Horn functions and then study in detail the special class of submodular functions. We give a one–to–one correspondence between submodular functions and partial preorders (reflexive and transitive binary relations), and in particular between the nondegenerate acyclic submodular functions and the partially ordered sets. This leads us to graph–theoretic characterizations of all minimum DNF representations of a submodular function and to show that the problem of recognizing submodular functions in DNF representation is in Co–NP. A production rule of a knowledge base is called ***essential*** if it is present in any prime knowledge base which is logically equivalent to the given one. In paper [32], we consider the problem of identification of essential rules, which constitutes a crucial part of the structural analysis of any knowledge base. It specifies the degree of freedom we have in constructing logically equivalent transformations of the base, by specifying the set of rules which must remain in place, and implicitly showing which rules could be replaced by other ones. A prime rule is called ***redundant*** if

it is not present in any irredundant prime knowledge base which is logically equivalent to the given one. The recognition of redundancy of a particular prime rule in a knowledge base will eliminate such rule from consideration in any future simplifications of the knowledge base. Paper [32] provides combinatorial characterization of essential and redundant rules of propositional Horn knowledge bases, and develops, whenever possible, efficient computational procedures for recognizing essentiality and redundancy of prime rules of Horn knowledge bases. In paper [21], we deal with the "pure" Horn functions, and prove for them a variant of the well–known Quine theorem which has a certain "non–expanding" property (every clause resulting from a consensus has a degree bounded by the maximum degree of a clause in the input CNF). Then we show how to use this result for generating all quadratic implicates of a given pure Horn function and prove that the generated algorithm has the best possible worst case complexity. Paper [23] is concerned with the variable deletion control set problem, the problem of finding a minimum cardinality set of variables whose deletion from the formula results in a DNF satisfying some prescribed property. Similar problems can be defined with respect to the fixation of variables or the deletion of terms in a DNF. In this paper, we investigate the complexity of such problems for a broad class of DNF properties. Paper [6] is concerned with positive boolean functions and the problem of their dualization. It is shown that every positive boolean function has a unique set of its true points, the set of so–called ***leftmost true points***, which includes all minimal true points of the function, and that the number of leftmost true points of a positive boolean function and of its dual are linearly related. A polynomial time dualization algorithm, which takes the set of leftmost true points as input and outputs the set of leftmost true points of the dual function, is presented. Duality is also the subject of paper [12], in which we consider dual subimplicants of positive boolean functions. Given a positive boolean function f and a subset Δ of its variables, we give a combinatorial condition characterizing the existence of a prime implicant of the boolean dual of f having the property that every variable in Δ appears in this implicant. We show that the recognition of this property is an NP–complete problem, suggesting an inherent computational difficulty of boolean dualization, independently of the size of the dual function. We also show that if the cardinality of Δ is bounded by a constant, then the recognition problem is polynomial. Finally, we have been studying a particular case of the satisfiability problem which strictly includes "nested satisfiability." In paper [39] we find a linear algorithm for this problem.

3. Location Problems

Location problems arise whenever a large set of potential sites for placing certain units is available and a selection must be made of the sites to be utilized. Such problems arise naturally in situations like placing warehouses, satellites, communication centers, military units, or emergency services. We have been specifically interested in location problems involving location of hubs, points of embarkation, staging areas, and other facilities of potential interest in network routing problems such as those arising at AMC or AFLC.

Our work on location problems in this project has emphasized three different directions: analysis of the objective function in location problems, clustering approaches to location problems, and solution of location problems in which judgements of closeness are made by users.

Traditionally in location theory (and elsewhere in problems of combinatorial optimization), the objective function is assumed a priori. Since there are so many

potential objective functions, it can be useful to try to specify reasonable conditions on an objective function, and choose an objective function which satisfies these conditions. Recently, Holzman [1990] took this approach. He tried to specify certain reasonable conditions which an objective function in a location problem should satisfy. Under some reasonable conditions, he showed that as long as the network had a tree structure, then the objective function was uniquely determined: It was to minimize the sums of squares of distances to the users. Vohra [1990] found different conditions which characterize this objective function on trees, and also found conditions which characterize the objective function of minimizing the sum of the distances to the users, again on trees. In trying to generalize Holzman's conditions to more general networks, we have discovered instead that the conditions are inconsistent and lead in all connected networks with cycles to an impossibility result. This work was begun in an earlier project, but has recently been improved and updated, with modified axioms and more detailed comparisons with other axiom systems proposed by Holzman. It is described in paper [40].

In many practical problems of location, we seek methods for clustering alternatives into groups. For instance, clustering methods have been used in developing solutions to a number of location problems of interest to AMC, specifically in locating (through the OADS model) U.S. hubs at Travis Air Force Base in California and Tinker Air Force Base in Oklahoma; in identifying good points of embarkation in deliberate planning models; in identifying staging areas for medical evacuations; and in identifying hubs for the defense courier system. We have investigated clustering methods, especially as they relate to location problems. Clustering methods are also relevant to the analysis of various practical problems of the Air Force which involve large amounts of data. These problems arise in such diverse contexts as early warning systems, detection of enemy positions, remote operations in space, cargo movement, "troubleshooting" in complex electronic systems, and forecasting. The data that arises is often noisy and unreliable, sometimes arising in hazardous or nuclear or chemically toxic environments, or under jamming, or just subject to great uncertainties. We can use clustering methods to detect patterns or to identify underlying causes. Clustering methods are also important in medicine (see for example Godehart [1990]), in genetics (see for example Arratia and Lander [1990]), and in the theory of social networks (see for example Johnsen [1989]). While developing clustering methods for location problems, we have kept other Air Force applications of these methods in mind. It is very rare for a clustering problem to have a polynomial–time algorithm for exact optimal clustering, due to the usual large number of possible clusterings. One of the few exceptions is due to Fisher [1958], who was one of the first to use consecutive partitions. He considered a one–dimensional clustering problem where the goal is to minimize the sum of squares, and proved that there exists a consecutive optimal p–partition. He asked for a generalization to more than one dimension. In paper [18], we provide such a generalization. We do this by studying partitions of integers, which are among the most widely studied combinatorial structures. A partition of a set N of n distinct numbers is called ***nested*** if there do not exist four numbers $a < b < c < d$ in N such that a and c are in one part while b and d are in another. A partition is called a ***p–partition*** if the number of parts is specified at p and a ***shape–partition*** if the sizes of the p parts are also specified. There are exponentially many p–partitions but only polynomially many nested p–partitions. Recently, Boros and Hammer showed that under certain partition–cost functions, an optimal p–partition is always nested. In paper [18], we give a general condition on the cost structure for which an optimal shape–partition is always nested. The results help to solve Fisher's problem.

Our work on clustering problems has led us to develop good algorithms for two clustering problems, known as average linkage clustering and divisive hierarchical clustering. See paper [25].

A number of approaches to location problems derive the solution from judgements of closeness. For instance, suppose that every user identifies which locations for facilities are "sufficiently close" to him or her. Then we seek a solution to the location problem which also assigns to each user a sufficiently close facility. Of some particular relevance here is the interval graph model, in which we start with judgements of closeness, assign to each element being judged a real interval, and take two intervals to overlap if and only if the corresponding elements are close; the real intervals are then used to define the clusters. All of this can be accomplished if and only if the graph whose vertices are the elements and whose edges correspond to closeness defines an ***interval graph.*** Interval graphs arise in numerous applications, including problems involving scheduling, transportation and communications, computer systems, ecosystems, foundations of computation, genetics, and seriation in the social sciences. See Fishburn [1985], Golumbic [1980], Trotter [1988,1992], and Roberts [1976,1978] for a summary of the literature of interval graphs and a discussion of many of their applications. We have been studying a special class of intersection graphs related to the interval graphs, where instead of real intervals we use intervals of boolean lattices (see Fishburn [1985]). These variants on the ordinary interval graphs led us to investigate ***bipartite covers*** of graphs, families of complete bipartite subgraphs whose edges cover the graph, to the notion of ***bipartite dimension*** $d(G)$ of a graph G, the minimum cardinality of a cover, and to the notion of ***bipartite degree*** $n(G)$ of G, the minimum over all covers of the maximum number of covering members incident to a vertex. In paper [31] we prove that $d(G)$ equals the boolean interval dimension of the irreflexive complement of G, identify minimal forbidden induced subgraphs for $d \leq 2$ and $n \leq 2$, and some of these subgraphs for more general $d \leq n$, and obtain results about $d(K_n)$ and $n(K_n)$.

Interval graphs are part of the more general class of graphs called ***perfect graphs*** that has a wide variety of important practical applications, including applications to important clustering and scheduling problems of various kinds. (See Fishburn [1985], Golumbic [1980], Opsut and Roberts [1981], and Roberts [1976, 1978] for references.) Notions of partitionability are important in some characterizations of perfect graphs (see Bland, Huang, and Trotter [1979]). In paper [9], we investigate known constructions of circular symmetric partitionable graphs and show that these constructions do not yield minimally imperfect graphs except for ***odd holes*** and ***antiholes***, odd chordless cycles with at least five vertices, and their complements, respectively. Moreover, we show that no partitionable graph with circular symmetry on an even number of vertices can be minimally imperfect. Results of Lovasz and Padberg entail that the class of partitionable graphs contains all the potential counterexamples to Berge's famous strong perfect graph conjecture (which asserts that the only minimally imperfect graphs are the odd holes and antiholes). In paper [3], we study a construction (due to Chvatal, Graham, Perold, and Whitesides) for partitionable graphs with circular symmetry and show that it produces no such counterexample. We also conjecture that every partitionable graph with circular symmetry can be generated by this construction, and give partial results in this direction.

In paper [10], we show that perfect graphs are kernel solvable, as was conjectured by Berge and Duchet in 1983. The converse statement was also

conjectured by Berge and Duchet, but remains open. In this direction, we prove that it is always possible to substitute some of the vertices of a non–perfect graph by cliques so that the resulting graph is not kernel solvable.

In paper [26], we study bull–free perfect graphs. A ***bull*** is a graph obtained by adding a pendant vertex at two vertices of a triangle. A graph is perfectly orderable if it admits an ordering such that the greedy sequential method applied to this ordering produces an optimal coloring for every induced subgraph. We investigate a conjecture due to Chvatal, which asserts that every bull–free graph with no hole or antihole should be perfectly orderable, and solve it partially. Our method is based on a partition lemma which lays out the structure of all bull–free weakly triangulated graphs.

A ***matrix*** of 0's and 1's is called ***perfect*** if the associated set packing polytope $P(A) = \{x: Ax \leq 1, 0 \leq x \leq 1\}$ is integral. Perfect matrices have many interesting properties and the perfectness of a 0,1 matrix is closely related to the perfectness of an associated graph. A ***matrix*** of 0's, 1's, and –1's is called ***perfect*** if the corresponding generalized set packing polytope $P(A) = \{x: Ax \leq 1-n(A), 0 \leq x \leq 1\}$ is integral, where $n(A)$ is the vector whose r^{th} component is the number of negative entries in row r of A. In paper [8], we provide a characterization of such perfect matrices in terms of an associated graph which one can build in $O(n^2m)$ time, where $m \times n$ is the size of the matrix. We also obtain an algorithm of the same time complexity, for testing the irreducibility of the corresponding generalized set packing polytope.

We have applied the theory of perfect graphs to some problems of game theory that are also relevant to design of efficient and reliable networks (since network design problems can be reduced in some cases to game theory problems — see for example McLean and Blair [1991]). A game can be defined by the set I of players and the set A of outcomes, and a ***coalition*** is then a subset of I. The ***core*** of a game is defined as the set of outcomes acceptable for all coalitions and it is probably the simplest and most natural concept of cooperative game theory. An ***effectivity function*** is a boolean function on the set $I \cup A$. An effectivity function is called ***stable*** if the core is not empty for any payoff function. The problem of characterizing stable effectivity functions seems, in general, very difficult. In paper [11], we apply a graph–theoretic approach to this problem. Using a graph based model, we obtain some necessary and sufficient conditions for stability in terms of perfect graphs, and we demonstrate that a conjecture by Berge and Duchet from 1983 is a special case of the considered problem of stability of effectivity functions. In paper [14], we note that some players may not like or know each other, so they cannot form a coalition. Let K be a fixed family of coalitions. The ***K–core*** is defined as the set of outcomes acceptable for all coalitions from K. The family K is called ***stable*** if the K–core is not empty for any normal form game. We prove that a family K of coalitions is stable if and only if K is a normal hypergraph.

A critical aspect in the study of perfect graphs is the determination of size of the largest independent (stable) set in a graph. In paper [22], we study the "struction" algorithm that has been widely used as a way of estimating the size of the largest independent set, the so–called ***independence number*** or ***stability number***. This algorithm creates a new graph whose stability number is one less than that of the original graph. By repeating the process enough times, an empty graph is obtained, and the stability number of the original graph is then equal to the the order of the final empty graph plus the number of iterations performed in order to

get that empty graph. We have implemented this algorithm and shown that it performs very well on several classes of graphs, such as triangulated and line graphs.

Returning to the approach to location and clustering problems based upon concepts of closeness, we note that closeness is often determined by whether or not two alternatives are within "threshold." The notion of threshold is the underlying motivation for the study of threshold graphs and threshold boolean functions. Threshold graphs were defined by Chvatal and Hammer [1977] and have since been applied to Guttman scaling in measurement and to synchronizing parallel processors, among other applications. Related to threshold graphs are threshold boolean functions, which are studied in paper [38]. A real-valued function G is said to be a ***separator*** of a boolean function f if for every boolean vector x, $G(x)$ is nonnegative if and only if $f(x) = 1$. If a boolean function f has a separator G that is affine in x, we call f a ***threshold function***. Polynomial time algorithms have been previously developed for checking if a given (monotone) boolean function is threshold and, if so, for computing the affine separator. In paper [38], we investigate the problem of computing a quadratic separator, when one exists. We also look at the problem of combinatorially characterizing the thresholdness and quadratic thresholdness of boolean functions. Threshold graphs are studied in paper [27]. Let $I(G)$ be the set of subsets X of the vertex set of graph G such that the subgraph of G induced by X is a threshold graph. If $I(G)$ is the independence system of a matroid, then G is called ***matrogenic***. We characterize the matroids that arise from matrogenic graphs.

4. Scheduling Problems

Many Air Force activities involve ***scheduling problems***. For instance, at AMC, scheduling problems arise in allocating loads to airplanes, assigning loads to points of embarkation and to routes, assigning crews to airplanes, and so on. Scheduling theory has long been a major area of interest in operations research, and there have been literally hundreds of papers written in the field. Some excellent references on the subject are the book by Baker [1974] and the volume of the Annals of Operations Research edited by Blazewicz, et al. [1986], as well as the survey articles by deWerra [1989], Lenstra and Rinnooy Kan [1978], and Lawler [1983]. However, the particular Air Force scheduling problems mentioned above have numerous complications which scheduling theory has not addressed. We have been investigating a variety of approaches to scheduling which take into account extra complications motivated by Air Force problems, in particular taking account of user preferences for schedules, finding schedules which meet performance standards such as those embodied in the UMMIPS priorities at AMC, and taking account of conflicting requests for schedules.

Scheduling problems involve assigning to each proposed "user" (person, piece of equipment, etc.) a (running or loading) time or location. There are usually two kinds of constraints in scheduling, ***precedence constraints*** (this use must precede this) and ***resource constraints*** (these two uses cannot overlap because they use the same resources or would together use more than the available resources). We have studied a specific scheduling problem under resource constraints, namely the so-called ***completion time variance minimization problem***. Here, there are n jobs to be scheduled on a single machine. For each job, its processing time is given, and the objective is to minimize the variance of the completion times of the jobs. In paper [4], we present a fully polynomial ϵ–approximation scheme for this problem with $O(n^3/\epsilon)$ time complexity. We do so by showing how this

scheduling problem is a special case of a more general problem known as the minimization of half–products problem. A half–product is a special kind of quadratic pseudo–boolean function. We show that while the minimization over the set of binary n–vectors for half–products is NP–complete, an ϵ–approximating solution can be found in polynomial time for any $\epsilon > 0$.

In many cases, the goal of a schedule is for items to be completed or to arrive at a given location by a certain time. For instance, AMC has developed a series of priorities or performance standards called UMMIPS for its schedules. Under UMMIPS, some highest priority items must reach their desired location within a short period of time, independent of cost, and there is a high "penalty" for not making the delivery on time. Lower priority items can arrive within a longer period of time and the penalty for missing the arrival time is lower. We have brought notions of desired arrival times, diverse performance standards, and varying penalties for missing desired arrival times, into the theory of scheduling. Motivated by these problems at AMC, we have formulated precisely a variety of scheduling problems under performance constraints. In the problems we have analyzed, a number of items (equipment, people) have to be moved from an origin to a destination. We assume that each item has a desired arrival time at the destination and that we are penalized in some way for missing that time. The penalty can be applied only for a late arrival or, more generally, for both late and early arrivals, perhaps in a different way. We assume that we can only take a certain number of items from origin to destination each time that we schedule a trip (say because we have only a limited number of seats on each plane and only a limited number of planes). Our goal is to minimize the total penalty. We also consider the complication that the items have different priorities or status or importance. This complication is specifically motivated by the UMMIPS priorities. If there are different priorities, the penalty for early or late arrival can depend upon the priority. In [46] we make this problem precise, formulate a variety of specific penalty functions, and summarize a variety of relevant papers in the literature. We note that the introduction of priorities adds a complication if we take into account the way we measure them. Namely, scales of measurement often have certain arbitrary choices (such as of unit or zero point). If we allow admissible transformations of scale, we should ask if the optimal solution to the scheduling problem, the solution which minimizes the penalty, remains unchanged. We observe that under some reasonable assumptions, it does not, and we give conditions under which it does. We emphasize analysis of the situation where the desired arrival time is the same for all items, and point out that, even here, we can be in the anomalous situation where an allowable change in the way we measure priorities changes the optimal schedule. The results have implications for how scheduling with priorities should be carried out.

In paper [45], we take the results in [46] one step further, analyzing scheduling problems in which not all items have the same desired arrival time. We give some general conditions under which a conclusion of optimality for a schedule is invariant under change of scale of the scale measuring priorities. In brief, these conditions require that the penalty increase with increasing priority and with increasing distance from the desired arrival time, but that early and late arrivals be treated equally and that specifically the penalty involves a linear function of distance from desired arrival time. We note that the conclusion is false under certain relaxations of these conditions, such as symmetric penalties for early and late arrival and quadratic functions of distance from desired arrival time. We show that the optimal solution to a variety of scheduling problems under performance constraints can be obtained by a simple greedy algorithm. We also

present surprising examples to show that this greedy algorithm does not attain optimality in all situations. In formulating the scheduling problem with earliness/lateness penalties and priorities that we investigate in this paper and paper [46], we have interacted with Alan Whisman at AMC and shared some of our ideas with him.

Paper [47] was motivated by papers [46] and [45], and makes the same kind of analysis for the widely studied problems of single machine scheduling. We consider the problem of finding the optimal schedule for jobs on a single machine when there are penalties for both late and early arrivals. We point out as in the earlier papers that if attention is paid to how certain parameters are measured, then a change of scale of measurement might lead to the anomalous situation where a schedule is optimal if these parameters are measured in one way, but not if they are measured in a different way that seems equally acceptable. We discuss conditions under which this anomaly is avoided.

The results in papers [46], [45], and [47] suggest looking more systematically at the meaningfulness of conclusions of optimality for problems of combinatorial optimization. A simple example will illustrate this. In the shortest path problem, if we transform each edge–weight w by a function of the form $d(w) = aw+b$, where $a > 0$, i.e., if we change both units and zero points, we can easily change the shortest path between two vertices. On the other hand, this is not the case with minimal spanning tree; the result follows from Kruskal's algorithm. In paper [48] and thesis [49], we analyze a variety of combinatorial optimization algorithms, and consider conditions under which conclusions of optimality do not change under admissible change of scale. The emphasis is on ordinal scales, scales where any monotone increasing transformation is admissible, and we have obtained some fundamental results connecting ordinal scales and greedy algorithms, results that can be stated in a surprisingly general, abstract setting.

As we have noted, scheduling problems are often analyzed in the context of of preferences stated by users. Suppose that user a and user b both wish an assignment at time or location x. Then there is a conflict (unless x has a large enough capacity to handle both, which is a special case which we have disregarded at first). In general, one can study such conflicts by considering a bipartite digraph D whose vertices are elements of two sets, the set S of users and the set T of times or locations, and which has an arc from user a to time (location) x if user a is willing to depart at time (from location) x. Then there is a corresponding graph G whose vertices are the users and which has an edge between users a and b if and only if there are arcs from both a and b to the same x. There is a rather extensive literature devoted to the study of the graph G, which is called the ***conflict graph*** corresponding to D. This concept of conflict graph arises in a variety of applications. For instance, in communications, S is a set of transmitters and T a set of receivers and there is an arc in D from a to x if a message sent at a can be received at x; the graph G represents conflict between transmitters. In coding, S is a set of codewords in a transmission alphabet, T a set of codewords in a receiving alphabet, and there is an arc in D from a to x if a word a can be received as a word x. The graph G represents confusability between codewords. Other applications arise in modelling of complex systems and in ecology, where the conflict graphs are called ***competition graphs***. Surveys of the extensive literature of the subject of these conflict/competition graphs can be found in the paper by Lundgren [1989] and in the theses by Kim [1988], Tesman [1989], and Wang [1991]. While there has been a considerable literature devoted to the ecological application

of conflict/competition graphs, we know of only several papers devoted to their applications to scheduling problems. One paper, by Raychaudhuri and Roberts [1985], lays out the general outlines of many applications. Another, by Hefner and Hintze [1990], follows on Raychaudhuri and Roberts and is based on a question arising from large naval communication networks. We have been developing the theory of conflict/competition graphs from the point of view of its scheduling applications. Specifically, as we have noted, conflict graphs arise from CCC networks. In this context, building on work begun in an earlier project, we have started in paper [62] to analyze some of the highly reliable network structures developed for computer and communication networks We have identified the structure of the resulting conflict graphs and have found structures for which it is easy to solve such scheduling problems as arise in frequency assignment.

The *competition number* of a graph G is the smallest k so that G together with k isolated vertices is a conflict/competition graph of an acyclic digraph. This number was introduced in 1978 by Roberts, who showed that the problem of its computation was equivalent to the problem of characterizing conflict/competition graphs of acyclic digraphs. Opsut [1982] showed that computation of this number was NP–complete. Based on an elimination algorithm developed by Parter and Rose for choosing the order of pivot points in Gaussian elimination, Roberts suggested in 1978 an elimination algorithm for computing the competition number. Opsut [1982] showed that this algorithm could overestimate the desired number. In paper [42], we have modified the elimination algorithm and showed that it correctly calculates the competition number for a large class of graphs. Using the original elimination algorithm, Roberts in 1978 found a formula for the competition number of a connected graph with no triangles, and this result has been widely used in the development of the theory of conflict/competition graphs. In paper [43], we have looked at the competition number of connected graphs with small numbers of triangles, and found exact solutions for the case where the graph has either one triangle or two triangles. The original elimination algorithm for Gaussian elimination was based on the idea of finding a so–called perfect elimination ordering for a triangulated graph. In paper [24], we study a generalization of perfect elimination orderings called domination elimination orderings. We show that graphs with the property that each induced subgraph has a domination elimination ordering (domination graphs) are related to formulas that can be reduced to formulas with a very simple structure. We also show that every HC–free graph, a graph having no induced "house" and no chordless cycle of length at least five, is a domination graph, and that every ordering produced by "maximum cardinality search" on these graphs is a domination elimination ordering.

A widely–studied scheduling problem is the job shop scheduling problem. This is a special case of a ***multiquadratic programming problem (MQP)***. An MQP is a problem of globally minimizing a quadratic function subject to quadratic equality and inequality constraints. It offers a powerful unification of several mathematical optimization problems. Besides job shop scheduling, other special cases of MQP are conventional quadratic programs and binary integer programs. Also, the more general problem of polynomial programming can be reduced to MQP. In paper [60], we have studied the exactness of a certain relaxation of MQP. For simplicity, suppose $f(x)$ is a quadratic map, i.e., each of its components is a (multivariate) quadratic function. Suppose we wish to determine whether $f(x) = b$ has a feasible solution for some right hand side b. This problem is NP–hard and hence not likely to entertain efficient solution procedures. However, the relaxation is to the problem of checking whether b lies in the convex hull of the image of f. This reduces to a convex programming problem

called the semidefinite programming problem (see below for definition), and there are efficient algorithms for its solution. We say that f is ***ICON*** if the image of f is convex. We have developed an explicit expression for the convex hulls of the images of quadratic maps and applied it to develop a characterization of ICON maps which is then employed to demonstrate that the ICON map detection problem is NP–hard for general quadratic maps. We consider two variations of the ICON property, namely, that f is convex for every linear subspace of the domain space and f is convex for every affine subspace of the domain space. For each of these properties, we develop an efficient polynomial algorithm for detecting if a quadratic map has this property.

As we have just noted, the analysis of the job shop scheduling problem and more generally MQP leads to an interest in semidefinite programming. In paper [59], we have explored this interest further. A real symmetric matrix is called ***positive semidefinite (PSD)*** if all of its eigenvalues are nonnegative. Given real symmetric matrices $Q_0, \ldots, Q_m$, define the matrix map $Q(x) = Q_0 + z_1Q_1 + \ldots + z_mQ_m$ and consider the set $G = \{x: Q(x) \text{ is PSD}\}$. Sets of this type are called ***spectrahedra***, they generalize polyhedra, and they involve eigenvalues (spectra). Linear optimization problems over spectrahedra are called ***semidefinite programs (SDP's)***. In paper [59], we have investigated some geometric properties of spectrahedra. We have characterized the faces of spectrahedra and have given expressions for spectrahedral cones. We have also characterized the conditions on the matrices $Q_0, \ldots, Q_m$ for the polyhedrality of spectrahedra and we have given a spectrahedral characterization of the convexity of a quartic multivariate polynomial.

In paper [58], we present a new and more complete duality for semidefinite programming, with the following features. The dual is an explicit semidefinite program, whose number of variables and the coefficient bitlengths are polynomial in those of the primal. If the primal is feasible, then it is bounded if and only if the dual is feasible. The duality gap, i.e., the difference between the primal and the dual objective function values, is zero whenever the primal is feasible and bounded. Also, in this case, the dual attains its optimum. The duality yields a precise Farkas Lemma for semidefinite feasibility systems, i.e., characterization of the infeasibility of a semidefinite inequality in terms of the feasibility of another polynomial size semidefinite inequality. Note that the standard duality for linear programming has all of the above features, but no such duality theory was previously known for semidefinite programming in general. We apply the dual to derive certain complexity results for semidefinite programming problems.

Quadratic knapsack problems arise in scheduling in various ways, for example when we are trying to pack a certain number of activities into a certain time period in an efficient way. In paper [37], we propose a heuristic algorithm and an exact branch and bound algorithm for the problem of maximizing a quadratic function in 0–1 variables which are subject to a linear inequality. The proposed algorithms aim at determining, at every step of the procedure, variables which can be fixed to values that are both feasible and optimal for certain subproblems. The algorithms make repeated use of the L_2–best linear approximation of the objective function and use for variable fixation conclusions derived Lagrangean relaxation, order relations and constraint pairing. We investigate the role of each of the three variable fixation techniques using computational tests. Extensive computational results are presented comparing the proposed heuristic and exact algorithm to previously known ones. The conclusions show that the solution produced by the proposed heuristic algorithm is, on the average, within 1% of the optimum and

that the number of vertices examined in the proposed exact algorithm is dramatically smaller than the number examined in published methods.

In Section 2, we discussed our approach to a Euclidean Steiner tree problem with obstacles. In papers [67,68] we have discussed a similar scheduling problem, a Euclidean traveling salesman problem with obstacles. Attempts to obtain a polynomial algorithm for vertices of k nested convex polygons, $k \geq 2$, and for $k = 2$ in particular, have so far been unsuccessful. However, if the inner convex polygon is an impenetrable obstacle, then a polynomial time algorithm is shown to exist, by transforming the problem into $O(m)$ shortest path problems on an acyclic digraph, where m is the number of obstacle vertices. The approach is generalized to arbitrary polygonal obstacles.

5. Solving Stochastic Integer Programming Problems

Many problems of the Air Force, and in particular those at AMC, have been approached by formulating large linear programs. For example, large LP models such as the Optimal Airlift Distribution System (OADS) and the Strategic Transportation Optimization and Routing Model (STORM) have played a major role in redefining AMC's routing system and in utilizing it. Many of the problems to which these models are applied require integer decision variables. One cannot have fractional aircraft, for example. Similar integer decision variables arise for instance when we assign tankers to bases, as for example in the Operational Support Airlift (OSA) basing model developed for AMC. Here, the variables are even 0–1 variables: Do we put tanker x at base y? Other integer programming problems of interest to the Air Force are described in the RAND Corporation report by Schank, et al. [1991].

The parameters (capacities, prices, technology coefficients) in the above–referenced problems are often random. We have worked on the development of theoretical methods for handling randomness when we add the additional complication that the decision variables are integers. Integer decision variables lead to integer programming while randomness of parameters leads to stochastic programming. There has been a great deal of research done on both of these topics. However, realistic models require both complications, and require a theory of *stochastic integer programming*. Because of its higher level of complication, results in stochastic integer programming are scarce.

Our investigation of stochastic integer programming has involved a major component aimed at developing the theory. We have developed this theory in directions motivated by the applied problems of reliability, location, and scheduling described above. Our emphasis has been on developing ways to handle reliability constraints and to maximize reliability and to produce methods which are computationally useful.

A part of our effort has been to summarize the most important and relevant results in the field of programming under probabilistic constraints and maximizing a probability under constraints. When the functioning of a system that is influenced by random effects has to be ensured by high reliability, then these methods offer powerful tools to achieve the goal. In paper [53], we analyze different problem formulations, convexity methods, and numerical solutions of problems of continuous and discrete variables. We also survey a variety of applications of the methods, for example concerning power system planning, electronic design, inventory control, water resources, reservoir system design, diet

and nutrition, animal feed, design of engineering structures, determination of optimal size of a runway at an airport, and financial planning.

A major accomplishment has been the completion of a 600 page book [54] on stochastic programming. It is the first comprehensive monograph written about the subject and its preparation was supported in its later stages by this grant. In addition to the material taken from the literature, the book contains quite a few new results. For example, we present a new way to use probability bounding techniques in probabilistic constrained stochastic programming. We show how a basis decomposition technique can be applied to the solution of multi–stage stochastic programming problems. We present laws of large numbers for random linear programs under more general and realistic assumptions than those published earlier in the literature. The book encompasses a wide range of ideas. Theories like logconcavity and moment problems are presented with algorithmic problem solutions and applications to power systems, water resources, finance, and other practical problems.

A good part of our effort has been concerned with programming under probabilistic constraints with discrete variables. If we have a linear programming problem where some of the right hand side values are random variables, then a powerful stochastic programming formulation is obtained by the combination of the simple recourse and probabilistic constrained model construction. In this problem, the sum of the expected penalized violations, which occur in the stochastic constraints, is added to the objective function while a probabilistic constraint guarantees that violations occur with a probability which is not greater than a prescribed level. In the paper [57] we solve the problem under the assumption that the random variables have a joint discrete distribution. As a special case, we obtain a solution for the probabilistic constrained stochastic programming problem for the discrete case and by this widen the applicability of this model, which has mainly been investigated in the continuous probability distribution case. The solution method uses the concept of a p–level efficient point (PLEP) introduced by Prékopa in 1990. The PLEP's are algorithmically generated and then a suitable cutting plane method carries out the rest of the job. The result in paper [57] allows for the solution of a stochastic network design problem which can be described as follows. Suppose that a network has random demands at the nodes, which may take positive and negative values (we call the demand supply if it is negative). At each node a generating capacity also exists, but it may not be enough to satisfy the local demand. In this case, the supply nodes assist the demand nodes to meet the demand entirely in the network. In the first approximation, we take the arc capacities constant, but an extension to the case where these are also random variables is not difficult. We want to invest into the generating capacities at the nodes and the arc capacities so that all demands should be satisfied on a prescribed, high reliability, level, at a minimum cost. The random demands are supposed to be dependent, discrete random variables. Numerical experimentation is underway, using the multivariate Poisson distribution as the joint distribution of the demand.

Due to the "curve of dimension," multi–stage stochastic programming problems cannot be solved exactly if the number of periods is not small enough. Already with five–stage problems, we have serious trouble. However, multi–stage stochastic programming problems provide us with important model constructions for various engineering, economic, finance, and other applications. Thus, we need techniques to obtain lower and upper bounds for the optimum value of the problem. Such a technique has been worked out and presented in paper [29]. The

technique uses a representation of a multivariate polyhedral function and ideas from the method of paper [51] (see below).

Until now, the solution of the "probabilistic constrained" stochastic programming model was solved only in the case of continuously distributed random variables. However, in many important applications, the random variables are discrete in nature. A solution algorithm for the problem has been worked out for this case, thereby filling a gap which limited the practical applicability of this model construction. The method, described in paper [2], works in such a way that we first enumerate the p–level efficient points (constituting the p–quantile of the probability distribution function involved) and then use a cutting plane method. A code in C language has also been created for the algorithm.

In paper [30], an efficient solution technique is worked out for the solution of the "simple recourse" problem. This is an LP where the right hand side values are random variables which are observed after the decision variables have been fixed. The problem is to find such optimal values for the decision variables which minimize the sum of the objective function of the LP and the expected penalty of the deviations between the deterministic left hand sides and the random right hand sides of the LP. Based on a formerly developed dual type algorithm, a numerically more efficient solution technique has been worked out together with its code. The technique uses the revised simplex method with produce form of the inverse. The code is written in C and various techniques are used to speed up the runs and improve on the precision. It seems that this method and code are the most efficient among the existing ones for the solution of the simple recourse problem.

Certain moment problems are existence problems in which we give conditions on finite or infinite sequences of numbers that are necessary and/or sufficient for the existence of a probability distribution of which these numbers are the moments. From the point of view of practical applications, a more important question is to find bounds on functionals of the unknown probability distribution under some moment information. Moments, at least some of them, are frequently easy to compute and the bounds that can be obtained on these grounds are frequently very good, in the sense that the lower and upper bounds for some value are close to each other. This means that these bounds can be used for approximation purposes as well. In paper [51], we present results pertaining to the second problem. Our functionals are expectations of higher order convex functions of random variables and probabilities of some events. We deal with the multivariate moment problems in the case of discrete probability distributions. Assuming the knowledge of a finite number of multivariate moments, we provide lower and upper bounds for probabilities and expectations of functions of the random variables involved. These functions obey higher order convexity formulated in terms of multivariate divided differences. As special cases, the multivariate Bonferroni inequalities are derived. The bounds presented are given as formulas as well as linear programming algorithms. The results are all fundamental in our analysis of stochastic linear programming.

Paper [44] is also concerned with upper bounds on probabilities, specifically the best upper bounds on the joint probability of a union of events, $P(A_1 \cup A_2 \cup \ldots \cup A_n)$ given knowledge of individual probabilities $P(A_i)$ and/or some joint probabilities $P(A_i A_j)$. We prove the sharpness of a bound given by Hunter, by formulating the bounding problem as a linear programming problem and then using the duality theorem to derive bounds.

In Section 2, we have already mentioned our work in paper [52] on the estimation of cause–effect relationships under noise. This paper solved the problem of estimating the weights of causes of an effect by formulating the problem as a stochastic programming problem and solving it by a dual type linear programing algorithm.

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RUTCOR

Grant Number F49620–93–1–0041

List of Publications Prepared Under the Grant

Note: RRR means RUTCOR Research Report

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57. Prékopa, A., and Vizvari, B., "Programming under Probabilistic Constraint with Discrete Variables," in preparation.

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59. Ramana, M.V., and Goldman, A.J., "Some Geometric Results in Semidefinite Programming," RRR 37–94, October 1994.

60. Ramana, M.V., and Goldman, A.J., "Quadratic Maps with Convex Images," RRR 36–94, October 1994.

61. Roberts, F.S., "On the Problem of Consistent Marking of a Graph," Linear Algebra and its Applications, 217 (1995), 255–263.

62. Roberts, F.S., and Wang, C., "Conflict Graphs of Highly Reliable

Networks," in preparation.

63. Shafer, G., Kogan, A., and Spirtes, P., "Generalization of the Tetrad Representation Theorem," Preliminary Papers of the Fifth International Workshop on Artificial Intelligence and Statistics, Ft. Lauderdale, FL, 1995, pp. 476–487. (RRR 26–93, October 1993.)

64. Winter, P., "Euclidean Steiner Minimal Trees for 3 Terminals in a Simple Polygon," Proc. of the 7th Canadian Conf. on Computational Geometry, 1995, pp. 247–253.

65. Winter, P., "Reductions for the Rectilinear Steiner Tree Problem," to appear in Networks.

66. Winter, P., "Small Euclidean Steiner Minimal Trees in Simple Polygons," submitted for publication.

67. Winter, P., and Shokoufandeh, A., "Shortest Tour Through Vertices of a Semi–convex Circular Channel," in preparation.

68. Winter, P., and Shokoufandeh, A., "Touring Vertices of a 2–dimensional Doughnut," in preparation.

69. Xu, S., "On Marked Graphs and Markable Graphs," preprint, RUTCOR, 1994.

70. Xu, S., "Min Cut in Planar Graphs," preprint, RUTCOR, 1994.

71. Xu, S., "Enumerating Cycle Bases," in preparation.

72. Xu, S., "Marked Graphs and Related Topics," Ph.D. thesis, RUTCOR, in preparation.

73. Xu, S., "Marked Graphs and 0–1 Matrices," in preparation.

74. Xu, S., "On Weighted Signed Graphs," preprint, RUTCOR, 1995.

75. Xu, S., "The Structure of 2–Connected Graphs," in preparation.

RUTCOR

FINAL TECHNICAL REPORT

TO: AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

DISCRETE, STOCHASTIC, AND OPTIMIZATION APPROACHES TO PROBLEMS OF NETWORKS AND SCHEDULING

GRANT NUMBER F49620–93–1–0041

Lectures Delivered

Peter L. Hammer

"Generalized Pure Literal Rule," invited lecture, ORSA/TIMS Meeting, San Francisco, November 1992.

"Recognizing q–Horn Formulas," invited lecture, ORSA/TIMS Meeting, San Francisco, November 1992.

"Computational Experiments with MAX–2–SAT," invited talk at APMOD'93, Budapest, January 1993.

"Boolean Functions and Graph Theory," two seminars at University of Puerto Rico, February 1993.

"On Universal Threshold Graphs," invited talk at Cambridge Combinatorial Conference, Cambridge, England, March 1993.

"Balancing Acyclic Data Flow Graphs," invited talk, ORSA/TIMS National Meeting, Chicago, May 1993.

"Graphs and Boolean Functions," invited talk at International Workshop and Conference on Discrete Mathematics, Chiang Mai, Thailand, October 1993.

"Boolean and Pseudo–Boolean Programming and Applications," minicourse, University of Paris, November 1993.

"Combinatorial Optimization in Structured Problems of Logic Minimization," invited talk at Workshop on Algorithmic Methods in Discrete Applied Mathematics, Oberwolfach, Germany, November 1993.

"Horn Functions: Structure and Minimization," seminar at National University of Singapore, December 1993.

"Boolean Methods in Artificial Intelligence," seminar at University of Hong Kong, December 1993.

"Graphs and Boolean Functions," seminar at Academia Sinica, Taipei, Taiwan, December 1993.

"Quadratic Unconstrained 0–1 Optimization," seminar at National Chiao–Tung University, Hsinchu, Taiwan, December 1993.

"Horn Functions: Structure and Minimization," seminar at National Tsing Hua University, Hsinchu, Taiwan, December 1993.

"Graph–based Methods for Horn Knowledge Compression," invited talk at Twenty–seventh Hawaii International Conference on Systems Sciences, University of Hawaii, January 1994.

"Knowledge Compression – Logic Minimization for Expert Systems," invited talk at IISF/ACM Japan International Symposium, Tokyo, March 1994.

"Logic Minimization for Horn Functions," seminar at Carnegie Mellon University, Pittsburgh, PA, April 1994.

"Knowledge Compression and Horn Function Minimization," invited talk at International Meeting of TIMS/ORSA, Anchorage, AK, June 1994.

"Decomposition of Positive Boolean Functions," invited talk at International Meeting of TIMS/ORSA, Anchorage, AK, June 1994.

"Logical Analysis of Data," invited talk at International Meeting of TIMS/ORSA, Detroit, MI, October 1994.

"Relevance of Variables and Rules in Rule–based Systems," invited talked at "Relevance" Symposium of the American Association of Artificial Intelligence, New Orleans, LA, November 1994.

"Essential and Redundant Rules in Horn Knowledge Bases," invited talk at Twenty–eighth Annual Hawaii International Conference on System Sciences, University of Hawaii, January 1995.

Fred S. Roberts

"The One–Way Street Problem," plenary lecture, Fall Meeting, Mathematical Association of America, Seaway Section, Cornell University, Ithaca, NY, November 1992.

"On Greedy and No–Hole Graph Coloring," colloquium talk, University of Tennessee, Knoxville, December 1992.

"The One–Way Street Problem," invited talk at DIMACS Conference on Discrete Math in the Schools, New Brunswick, NJ, January 1993.

"Innovative Curricula in the Mathematical Sciences for the 90's and Beyond," panel presentation, Conference on Graduate Programs in the Mathematical Sciences for the 90's and Beyond, Clemson University, Clemson, South Carolina, April 1993.

"Sturdy Networks," invited talk, American Mathematical Society Meeting, Washington, DC, April 1993.

"Sturdy Networks," plenary talk, Conference on Graphs and Matrices, Boulder, Colorado, May 1993.

"Meaningless Statements," colloquium talk, University of Pittsburgh, May 1993.

"Meaningfulness of Ordinal Comparisons for General Order Relational Systems," plenary lecture at International Conference on Ordinal Data Analysis, University of Massachusetts, Amherst, October 1993.

"Choosability and Amenability in Graph Coloring," plenary lecture at New York Academy of Sciences Graph Theory Day 26, Bard College, Annandale–on–Hudson, New York, November 1993.

"Meaningless Statements," colloquium talk, GERAD – Ecole des Hautes Etudes, University of Montreal, December 1993.

"From Garbage to Rainbows: The Many Applications of Graph Coloring," colloquium talk, University of Colorado, Denver, February 1994.

"Meaningless Statements," colloquium talk, Dartmouth College, Hanover, NH, May 1994.

"Graphs, Garbage, and a Pollution Solution," Cresskill High School, Cresskill, NJ, June 1994.

"Traffic Lights, Fleets, Mobile Radio Telephones, and Rainbows: The Many Applications of Modern Applied Mathematics," Ocean Twp. High School, Oakhurst, NJ, January 1995.

"Competition Graphs with a Small Number of Triangles," invited talk at American Mathematical Society Meeting, Orlando, FL, March 1995.

"Graphs, Garbage, and Pollution Solution," Math Club talk, Livingston High School, Livingston, NJ, March 1995.

Endre Boros

"Generalized Pure Literal Rule," invited talk, ORSA/TIMS meeting, San Francisco, November 1992.

"Recognizing q–Horn Formulae," invited talk, ORSA/TIMS meeting, San Francisco, November 1992.

"Computational Experiments with MAX–2–SAT," organizer of eight sessions on integer programming at APMOD'93, Budapest, January 1993.

"An Exact Algorithm for SAT with Computational Results," invited talk at DIMACS Workshop on Solving Hard Combinatorial Optimization Problems," New Brunswick, NJ, March–April 1993.

"Unconstrained Quadratic 0–1 Programming with Applications in Image Processing and Clustering," invited talk at DIMACS Workshop on Partitioning Data Sets, with Applications to Psychology, Vision, and Target Tracking, New Brunswick, NJ, April 1993.

"Quadratic 0–1 Programming Applied to QAP," invited talk at DIMACS Workshop on Quadratic Assignment and Related Problems, New Brunswick, NJ, May 1993.

"Balancing Acyclic Data Flow Graphs," invited talk at TIMS/ORSA National Meeting, Chicago, May 1993.

"Persistency Results in SAT and MAX–SAT Problems," invited lecture at workshop on Algorithmic Methods in Discrete Mathematics, Oberwolfach, Germany, October 1993.

"Algorithmic Results for SAT and MAX–SAT," three invited lectures at Eotvos Lorand University, Budapest, Hungary, November 1993.

"Generalized Pure Literal Rule and its Variants," invited talk at Third International Conference on Mathematics and Artificial Intelligence, Ft. Lauderdale, FL, January 1994.

"Perfect Graphs are Kernel Solvable," invited talk in Second Hungarian–American Combinatorial Optimization Workshop, Budapest, Hungary, May 1994.

"Dual Subimplicants of Positive Boolean Functions," invited talk at TIMS XXXII meeting, Anchorage, AK, June 1994.

"Algorithmic Results for SAT and MAX–SAT," invited talk at 15th International Symposium on Mathematical Programming, Ann Arbor, MI, August 1994.

<u>Andras Prékopa</u>

"Estimation of Cause–Effect Relationship under Noise," invited talk at IFIP Conference on Stochastic Programming, Visegrad, Hungary, March 1993.

"The Multivariate Discrete Moment Problem and its Use in Stochastic Programming," invited talk at Conference on Approximations in Stochastic Programming, IIASA, Laxenburg, Austria, July 1993.

"New, Efficient Bounds on Reliability of Series–Parallel Systems," TIMS/ORSA Joint National Meeting, Boston, MA, April 1994.

"On a Dual Method for a Specially Structured Linear Programming Problem," invited talk at International Conference on Stochastic Programming, Nahariya, Israel, June 1994.

<u>Alex Kogan</u>

"Optimal Compression of Propositional Horn Knowledge Bases," invited talk at Siemens Corporate Research, Princeton, NJ, November 1993.

"Knowledge Compression — Logic Minimization for Expert Systems," invited talk at 4th International Symposium on Artificial Intelligence and

Mathematics, Ft. Lauderdale, FL, January 1994.

"Decomposability of Partially Defined Boolean Functions," invited talk, Graduate School of Business, Columbia University, New York, NY, March 1994.

"On the Essential Test Sets of Discrete Matrices," invited talk at TIMS International Meeting, Anchorage, AK, June 1994.

"A Generalization of the Tetrad Representation Theorem," invited talk at Fifth International Workshop on Artificial Intelligence and Mathematics, Ft. Lauderdale, FL, January 1995.

Aleksandar Pekec

"On the Scheduling Problem with Earliness/Tardiness Penalties: When is an Optimal Solution Not Optimal?", invited talk at second International Colloquium on Graphs and Optimization, Loeche les Bans, Switzerland, August 1994.

"On the Scheduling Problem with Earliness/Tardiness Penalties: When is an Optimal Solution Not Optimal?", special lecture at Technical University of Graz, Graz, Austria, September 1994.

"On the Scheduling Problem with Earliness/Tardiness Penalties: When is an Optimal Solution Not Optimal?", contributed talk, ORSA/TIMS National Meeting, Detroit, MI, October 1994.

"On the Scheduling Problem with Earliness/Tardiness Penalties: When is an Optimal Solution Not Optimal?", seminar talk at SUNY, Stony Brook, December 1994.

"Applications of Measurement Theory to Combinatorial Optimization," invited lecture at Croatian Operations Research Society, Zagreb, Croatia, January 1995.

"Applications of Measurement Theory to Combinatorial Optimization," seminar talk, University of Colorado, Denver, February 1995.

"Limitations on Conclusions from Combinatorial Optimization Models," seminar talk, Carnegie Mellon University, Pittsburgh, PA, March 1995.

"A Winning Strategy for the Ramsey Graph Game," contributed talk at 26th Southeastern Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, March 1995.

Shaoji Xu

"On Weighted Signed Graphs," contributed talk at 26th Southeastern Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, March 1995.

Participants in
RUTCOR Project on "Discrete, Stochastic, and Optimization Approaches to Problems of Networks and Scheduling"

Faculty

Peter Hammer (Principal Investigator)

Fred Roberts (Principal Investigator)

Endre Boros

Andras Prékopa

Pierre Hansen (associate)

Yves Crama (associate)

Alex Kogan (associate)

Frederic Maffray (associate)

Ilya Muchnik (associate)

Pawel Winter (associate)

Postdoctoral Fellow

Motakuri Ramana

Graduate Students

Tamas Badics

Mario Cordova

Sheng Li

Jian–min Long

Hans Mielke

Aleksandar Pekec

Lorant Porkolab

Shaoji Xu